Fabbrica della Conoscenza

XIV Forum Internazionale di Studi

Le Vie dei Mercanti

Carmine Gambardella

WORLD HERITAGE and DEGRADATION

Smart Design, Planning and Technologies

La Scuola di Pitagora editrice

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Collana fondata e diretta da Carmine Gambardella

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Conference topics:

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WORLD HERITAGE and DEGRADATION
Smart Design, Planning and Technologies
Le Vie dei Mercanti
XIV Forum Internazionale di Studi

Napoli | Capri
16 - 17 - 18 June 2016

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Peer review

Scholars has been invited to submit researches on theoretical and methodological aspects related to Smart Design, Planning and Technologies, and show real applications and experiences carried out on this themes.
Based on blind peer review, abstracts has been accepted, conditionally accepted, or rejected.
Authors of accepted and conditionally accepted papers has been invited to submit full papers. These has been again peer-reviewed and selected for the oral session and publication, or only for the publication in the conference proceedings.

Conference report

300 abstracts received from:

Albania, Benin, Belgium, Bosnia and Herzegovina, California, Chile, China, Cipro, Cuba, Egypt, France, Germany, Italy, Japan, Jordan, Kosovo, Malta, Massachusetts, Michigan, New Jersey, New York, New Zealand, Poland, Portugal, Russia, Slovakia, Spain, Tunisia, Texas, Turkey.

More than 550 authors involved.
212 papers published.

Preface

The theme of the XIV Forum “Le Vie dei Mercanti” is an international discussion on the disciplines of architecture, design and landscape through the presentation of research and operational projects on the conservation and valorisation of World Heritage and “smart” regeneration of degradation, with analyses and proposals ranging from the design at all scales, to architectural assets, the territory, infrastructures and the landscape. Academics, along with professionals who have a role in the governing, managing and controlling of public agencies, institutions and the business world are invited to submit papers related to design objects, architecture and landscapes. This is with the aim of conserving and recovering, valorising and regenerating, managing and designing (or re-designing), for the more general improvement of the quality of life, in an innovative and contemporary relationship between man and the environment, through “beauty”, while respecting the history, traditions, identity and principles of sustainable development, as well as being attentive to the needs of our and future generations. Internet of Everything, smart design, planning and technologies, building information modelling, in this age of globalization, have become operational tools – that alongside the traditional ones of the profession – for the protection and promotion of the World Heritage, are considered as well as shared by the whole of Humanity, and the regeneration of the degradation and the “Minor Heritage”, in all aspects, and as contemplated by the UNESCO Conventions on tangible and intangible assets and the European Landscape Convention. The event aims to create a critical transversal dialogue, open to cultural and “unlimited” influences, in a logic of integration between the skills that extends, and is not limited, to the following disciplines: anthropology, architecture, archaeology, history art, cultural geography, design, ethnology and folklore, economy, history, landscape, museum management, philosophy and political science, urban history and sociology, cultural tourism, planning and integrated management. The location is exceptional. Campania, with six sites included in the World Heritage List, two UNESCO Man and Biospheres, two sites on the List of Intangible Heritage, is one of the richest regions in the world for cultural and landscape heritage.

Carmine Gambardella

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3D modeling of Pompeii amphitheater. Preliminary critical considerations

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Abstract
This abstract continues the discussion of the research presented at the last year's Forum about some important activities carried out within the archaeological site of Pompeii. In particular, the study introduces survey, analysis and modeling activities performed for the amphitheatre, combining the themes of knowledge of a cultural site of great value with the aspects of the conservation and safeguarding of degraded heritage.
The *Spectacula* of Pompeii, erected shortly after 80 B.C., is one of the most homogeneous buildings and better dating of Pompeii through the inscription that commemorates the construction (C.I.L., x, 852). The amphitheater of Pompeii, suitable for performances held there but not monumental, is the evolution of the architectural type of the amphitheater from false wooden construction to stable stone structure. Not changed nor extended by the consolidation restorations made after the earthquake of 62 A.D., the Pompeian amphitheater for its age is the oldest amphitheater architecture building.
The complex spatial configuration of both elevation elements and underground spaces has allowed some significant geometrical and critical considerations that frame the construction techniques, as well as the following consolidation and restoration works. Specifically, the geometric analysis involved the identification of the planimetric shape, the slope of the stands and tiers, as well as assessments on the three-dimensional configuration of the auditorium space.

Keywords: amphitheater, Pompeii, survey, analysis, 3D modeling

1. Introduction (Carmine Gambardella)
This monographic study continues the research presented at the previous Forum focused on a critical comparison between the various technical and scientific integrated digital survey steps applied to three case studies within the boundary of the archaeological site of Pompeii - the Amphitheatre, the Villa dei Misteri and the Torre di Mercurio - which are emblematic for construction and excavation dating, for historical and artistic importance, for material composition, for architectural morphology.

Fig. 1: Pompeii amphitheatre: panoramic view.

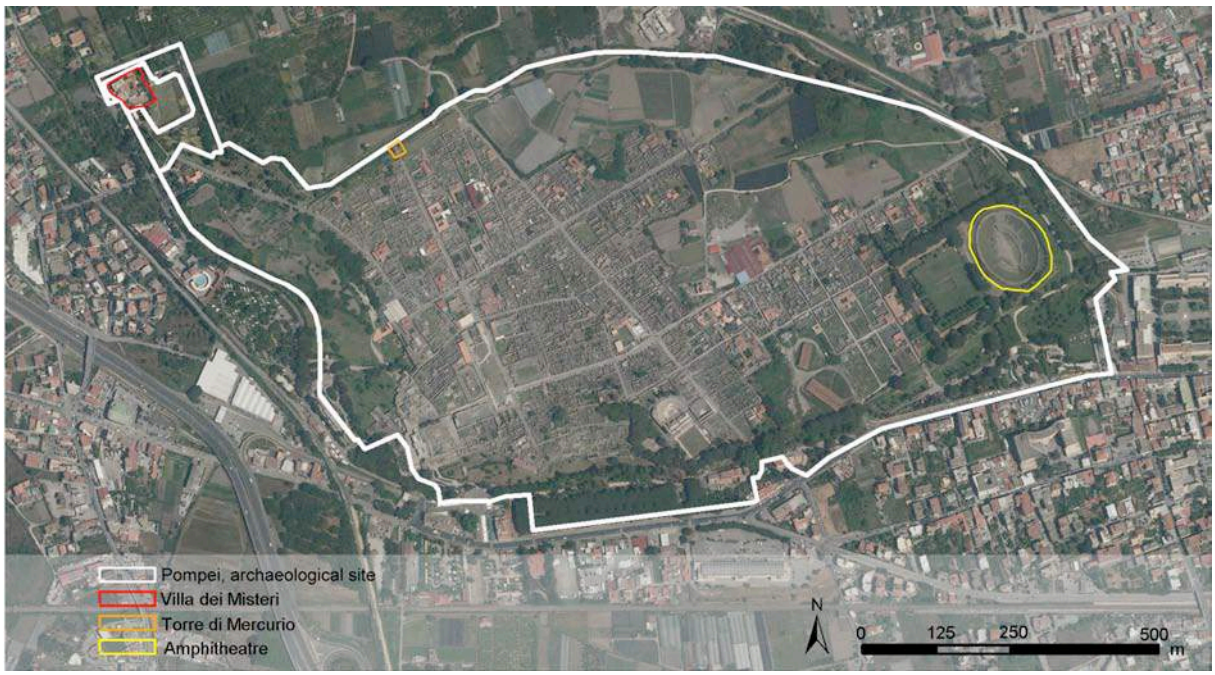

Fig. 2: Three case studies within the boundary of the archaeological site of Pompeii.

Fig. 3: Pompeii amphitheatre: underground ambulatory, main entrance, auditorium during 3D laser scanning activities and topographical survey by total station.

In particular, attention is directed exclusively to the amphitheater for which critical remarks were made in order to its configurative geometry on surveyed points with a considerable accuracy. This building was the subject of the integrated digital survey by applying 3D laser scanning sensors, GNSS topographic and close-range photogrammetric terrestrial and by drone aimed at the geometric and morphological characterization of each architecture. In particular, whereas the Amphitheatre is a large concave building, walkable along concentric paths on three significant levels at least, the survey was structured: to divide the building into homogeneous architectural sectors, corresponding to the same macro-scanning projects; to use two laser scanners Faro CAM2 and one total station. From the 3d model, following the phase of post processing of the data acquired as described above, useful information to intercept significant points of the amphitheater were extracted in order to understand, to learn and to investigate the complex geometry of the auditorium, the direction of the stairs crossing and reaching the different parts of the theatre space, such as the shape of the corridor below the auditorium itself; and also it is important to study the auditorium's slope in relation to the visibility inside the different parts of the theatre, or to study the trend of the tiers according to the problems of cutting of stones and other materials used for the construction. As it regards the planimetric shape of the amphitheater built from the outer wall from which then starts the construction of the internal parts of the auditorium, layout tracking of the external shape was dependent on the tracing of the geometry of successive internal parts. In fact, the tracking of the ellipse is much more laborious although operationally, especially for large curves, presents problems similar to those of the oval. The oval has however the advantage to be simply described by concentric curves (substantial problem for the bleachers) as well as a simplification in the preparation of stone that constitute the structure.
The above mentioned steps of analysis and knowledge of the amphitheater, from the 3d model surveyed through the extraction and analysis of significant horizontal and vertical sections, differ from experiences in other contexts and described by the scientific literature. This method does not limit the

study to interior and exterior stereometry of the building, to the simple form derived from projections on a plane, but deepening the study of the formal complexity of an architectural product declined in a multidimensional system for knowledge, protection, structural behavior and forecasting scenarios for recovery and modification of cultural heritage.

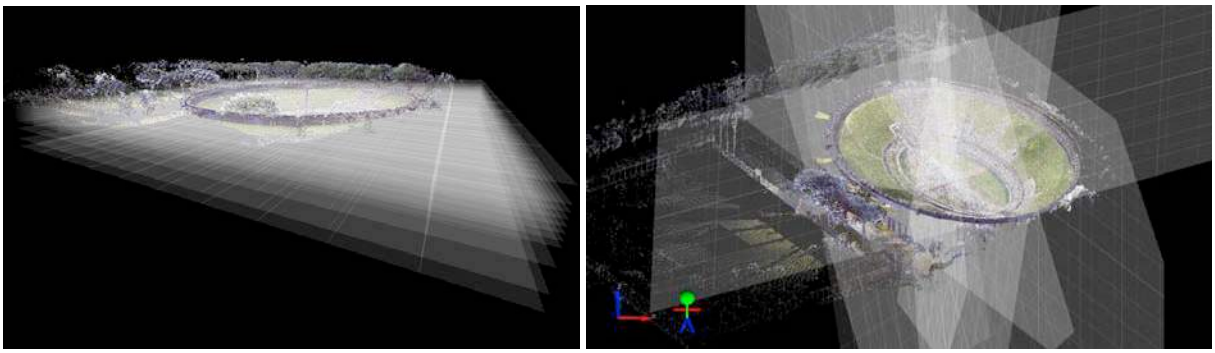

Fig. 4: 3D laser scanner model sectioned with horizontal plans (left) and vertical ones (right).

2. The *Spectacula* from wooden rectangle of stone ellipse (Nicola Pisacane)

The amphitheatre of Pompeii was built in the period immediately following the dictatorship of Sulla (82-79 b.C.), which put an end to the social upheaval due to civil wars, and in particular in conjunction with the foundation of numerous colonies of veterans. The architectural style of the amphitheatre, unknown to the Hellenic world, was introduced in the Roman Empire in order to give to colonies a permanent space for gladiator fights that, as events celebrated in the occasion of funerals, had become usual performances. The exponential development of gladiatorial games was recorded in Campania earlier than in other regions. From Campania, in fact, come the first evidence of stable masonry amphitheatres in Cuma, Capua, Pozzuoli and Pompeii dating from the middle of the second century b.C. and built directly on the ground and lying on the side of a slope, according to the logic of the Greek theaters. In Campania, where, as mentioned, the *ludi gladiatorii* (gladiatorial games) were perfected, there was the evolution of the architectural type of the amphitheatre from a mobile wooden construction to a stable structure in stone, of which the *Pompeii Spectacula* is for its age the oldest amphitheatre built in masonry. If in Campania are built amphitheaters stable in stone, in other regions, depending on availability of materials which offered the territory, the availability of money and the importance and solemnity of the gladiatorial games, they continued to make wooden and/or masonry amphitheatres. There are socio-economic causes that favoured the experimental building of the amphitheatre type in the Campania more than elsewhere, particularly in some of the military colonies and Romanized city, as Capua, Cuma, Pozzuoli and Pompeii which enjoy a strong cultural identity and of historical importance that has its roots in the Greek period and until the end of the fifth century. The commercial momentum and economic recovery of these cities by the end of the second century b.C. depended in part by the activity of "*negotiatores*", merchants belonging to the Roman ruling class that in the wake of Roman rule in these territories, based flourishing commercial activity in the eastern Mediterranean succeeding in some cases to monopolize trade with Italy; in part by the strategic geographical location of these centers crossed by all the main networks, such as the *Via Appia* linking Rome to Brindisi (the most important port of the whole of Greek and eastern area) passing through Capua, the *Via Latina* that was grafted to Casilinum, the *Via Popilia* that was inserted just through East direction from Capua, and the *Via consularis Puteolis-Capuam*, which linked Capua to Puteoli, the *Via Puteolis-Neapolim*. Among these, Pompei, Cuma and Sinuessa stand out among the other Campanian centers also important ports for harbor and commercial activities. The flourishing economic and commercial recovery of these centers favored the resumption of cultural activity. It is through cultural activities that Rome, which is faced with the difficulty of managing the strong historical identity of these military colonies and cities Romanized, manages to maintain control using for political purposes gladiator games and allowing the creation of an appropriate space , the amphitheater, for their implementation.

The term ‘amphitheater’, appears for the first time in imperial times, long after the appearance of the building that was built for the games gladiatorial defined *Spectacula*, as stated in the famous inscription that commemorates the construction. With amphitheater define those buildings to the Roman creation oval/elliptical plant that develop from the end of the second century b.C., reaching a monumental appearance during the imperial age and hosting gladiator fights (*munera*) and animals (*venationes*). Define the genesis of the amphitheater shape is not easy, mainly because of the lack of documentation on the transition from temporary What to do in the *Forum Romanum* wood, the oval/elliptical canonical stable of masonry buildings. Therefore, the *Forum Romanum* is the geometric prototype which should emanate the amphitheater form that seems to originate from a progressive

adaptation of rectangular areas, in the context of civic holes, for strictly functional nature motifs. The corners in the rectangular spaces away the action that takes place inside the amphitheater by the spectators; cutting or smoothing out the corners comes naturally to softer shape of the ellipse/oval, which might be defined as an elongated circle, or a circle with a tendency to linearity that best suits the diverse nature of events performed in the amphitheater, the shows for which it prefers a centric arena, and processions that require, however, a linear development of space. In general, in the amphitheater of Pompeii, you can seize the start of the operation mode of the arenas: the lack of an external porch; the stairs to the *summa cavea* resting outside and not set in the thick of the auditorium; the upper gallery limited to a series of boxes and of small doors arched; the lack of underground arena reserved more gladiatorial duels that hunts fairs and, finally, the ellipse of the auditorium built as a duplication of the auditorium of a theater. The verification of the elliptical amphitheater floor plan, already partly exposed in last year's paper, continued and extended to multiple horizontal plane sections of the amphitheater and exposed in this essay. The changes and the transformations that the building had over the centuries, suggested the research on the determination of the amphitheater's shape in several points. Specifically, the planimetric shape was analyzed from sections at different heights, so that from these sections the investigation could continue on vertical structures. The geometric investigations on vertical walls was possible because most of them, is original within specific architectural details that were lost due the eruption, or were looted in the years after AD 79 because the summit parts of 'Amphitheatre were the only ones to remain exposed after the catastrophe. These assessments are also confirmed in many historical iconographies. Among the first images of the Amphitheatre after the archaeological excavation, there are some gouaches representing the building, from "*Via Castricio*" (at the corner of The *Palaestra*). The graphical technique used and image purposes (they were marketed as souvenirs for visitors to the excavations) allow to have a true representation of the scene at the time, less than marginal details that were used to give a picturesque feel to the ruins. In gouaches reference, in fact, we see the double staircase, and the system of perimeter arches, partly ruined. The upper part of the *cavea* is largely non-existent precisely because those structures were the most slender, and were not fully covered by the eruption. This summit area and perimeter arches were restored in the second half of the twentieth century. The most suitable method for the elliptical shape's verification was Blaise Pascal's theorem, applied to five horizontal cross-sections of the auditorium at 14.0m, 18.9m, 23.0m and 28.0m and of hypogeum level at 17.0m over the WGS 84 model, and obtained crossing the mesh model of the amphitheatre, made from point clouds data. The theorem was published in 1640 by Blasie Pascal into the "*Essay pour les coniques*" dedicated to the principles and graphical results of any parabola, ellipse or hyperbole "*d'une manière plus universelle qu'à l'ordinaire*". In particular, this theorem defines the mystical properties of any hexagon inscribed in a conic.

Both the stands' planimetric system, and the four branches' shape of the underground corridor were analysed by choosing six random points not too close together, along the cross-section profile. Compared to these points a hexagon was built for each horizontal section. These hexagons should not necessarily be convex, as Leibniz pointed out in 1676 in his note to the theorem *(De Exagrammo mystico et conico)*, but the sides joining its vertices will also intersect each other.

For all the above five sections occurred the alignment of the three intersection points of the hexagon sides, which join two by two. The sections therefore are elliptical. Subsequent investigation steps allowed the determination of the points, and significant elements of the ellipses. In particular, the use of geometrical homology, and the application of the principles of planar polarity allowed the determination of the center of the ellipses. The same projective rules also allowed the identification of major and minor axes of the ellipse, and from these the foci (focal points) were determined for each of the elliptical sections.

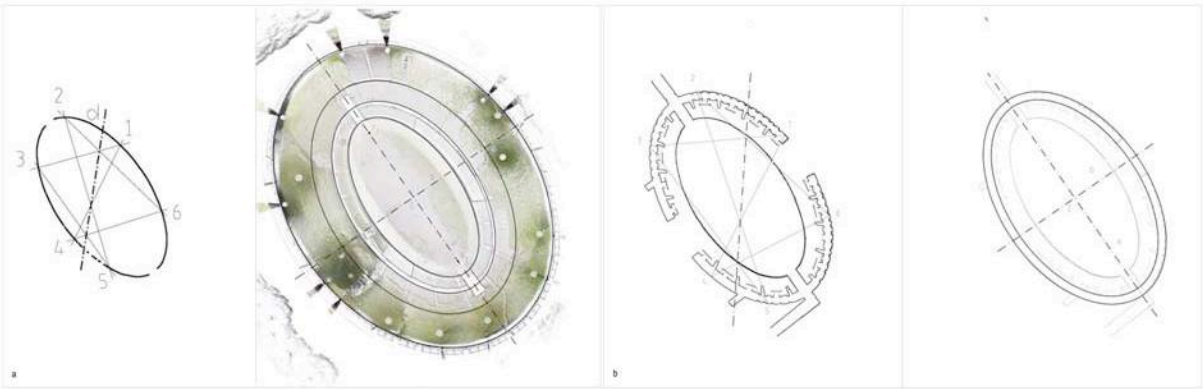

Fig. 5: Application of the Pascal's theorem and main elements of auditorium (left) and underground spaces (right).is important for our growth.

3. An integrated analysis of the amphitheater from the underground spaces to the auditorium (Alessandra Avella)

The amphitheater of Pompeii was built laying off center at the south-eastern boundary of the archaeological site near the fortified walls, specifically in an area of the ancient city outlying and still not occupied by buildings in the years when was erected. The building, therefore, uses the eastern corner of the city walls, reinforced inside by the *agger* (embankment), and is obtained by half in the embankment. The principles of the ancient method of consolidation by the *agger*, are in a way adapted to the needs of the construction of the amphitheater and to the rational use of what the environment provides. The construction on the embankment of the amphitheatres is not typologically earlier than masonry construction. In relation to the topographical situation and it means that you could have for the realization of amphitheater architecture was chosen one form or the other. It is for this reason that there are amphitheaters on the embankment belonging to the second century AD and masonry amphitheatres belonging to the same age. In general, the amphitheaters that belong to military contexts were built in most cases on the embankment, while those that belong to urban contexts were built mainly of stone.

The location off center of the amphitheater of Pompeii in the corner of the fortified walls, on one side facilitated the inflow and outflow of spectators on performance days, through the nearby 'Nocera' gate, on the other enabled a considerable saving in building costs, exploiting for the substructure of the auditorium a large tract consists of the embankment. At the same time, the construction on the embankment means that, of the two access archways to the arena, located at the end of the major axis, only the one to North side can follow in a straight line the same axis, while the other one to South side must, because of the fortified walls, bending at a right angle to the West. These access archways, 5 m wide, are made in *opus incertum* and in the typical reticulated dating back to Silla, paved and with a considerable slope in order to bridge the height difference between the ground level and the arena, realized through the removal of a large amount of terrain. The arena and the exterior of the building are connected by a narrow passage, which is accessed from the Libitinense gate on the western side at the minor axis of the ellipse. This gate was used to transport the dead out of the arena. When the arena is excavated to a much lower level than the surrounding level, the exterior of the monument is of course more modest. This does not occur in Pompeii where the embankment is supported for half by the city wall, and for the other half by a circular wall reinforced by masonry supports, connected to one another by 62 stone blind arcades, with walls made of *opus incertum* and in the typical reticulated dating back to Silla that give the monument a monumentality and a dynamic movement that is not always located within the static and triumphant magnificence of the amphitheatres of the imperial age.

On the free half of the ellipse two double flight stairs and two simple stairs, leaning against the circular wall at the south-west and north-east side of the amphitheater, leading to a terrace placed at the same height of the city walls, from which by 40 gates you can reach the inside of the amphitheater. The seats reserved for spectators were divided into many horizontal sections called *maeniana*. Between the first *maenianum* and the arena ran all the way around, like a huge ring, the *podium* or terrace intended to support the seats reserved to the authorities; the front wall of the *podium* went up and in an arena perpendicularly to a height of about two meters and was decorated with frescoes, today lost, depicting fights between gladiators. The different *maeniana* are separated from each other in a horizontal direction through open passages, called *praecinctiones*, in the vertical direction through walls, called *baltei*, that have a similar appearance to that of the front wall of the *podium*.

Between the first and second *maenianum* you could be accessed through the external stairs that divide the amphitheater into wedges. The *ima cavea* is divided into 18 wedges and includes 5 steps files; these steps are interrupted, in the middle of the longer sides of the ellipse, by two large terraces with 4 shallow and wide steps for the authorities chairs; on the east side (the corresponding place on the other side is destroyed), the second of the four steps is interrupted in the middle by a space of 3 meters reserved for magistrates who presided over the games and those that offered the show; the *media cavea* is divided into 20 wedges and twelve files of steps; the *summa cavea* was divided into 40 wedges with 18 steps to the top of which are recognizable remains of a wall in *opus incertum*, perhaps a remnant of accommodation in this part of the *cavea*. The bleachers currently visible, made by original stone material, had a decreasing slope from *ima cavea*, reserved for important people of the city, to the *media cavea* reserved to the people and the *summa cavea* reserved for women. Finally, above the upper outer ring remains are recognizable in *opera vittata* of an ambulatory which must be recognized *maenianum summum* reserved for slaves (servants), with only standing room.

In order to define the surface that best describes the shape of the auditorium, an analysis was conducted oriented towards the determination of the geometric rule that presides over the auditorium's partition in "wedges", through the system of the flights' stands.

This study was developed on the point cloud model of the northeastern sector, because it is the only original and fully preserved by the *ima-cavea summa-cavea*. This is documented fortunately in some prints and photographs of the nineteenth century, because most the draftsmen and photographers, choosing the frame with the cavea in the foreground and Mount Vesuvius in the background, had to be

placed on the southern stands. In this way, the sector of the northeast appeared framed in all the images. The comparison between the point cloud model and historical images (especially those by Sommer and Alinari) shows that this sector is intact from the early nineteenth century. The conservation of the Amphitheatre of Pompeii, enabled the investigation on the ellipsis's axes direction, which identify the wedges of the parterre limited to the northern quarter. This original auditorium's system allowed the verification of the flights' direction, projected on the horizontal plane.
These lines, traced for the average points of the ramps in best condition, off on them segments of similar length which radiate from the center of the archway on the elliptical wall at the top of the building until they reach the minor axis common to all the ellipses that define the stands, so for all forty equidistant passage. The rule that these lines follow, refers to the cycloid's construction, but it also refers to the positions that the writing arm follow while moving the elliptical compasses, described by Guidobaldo del Monte. This tool is described into text in two volumes "*Planisphaerium universalium Theorica*" (1579), which illustrates the problem of the projection of the celestial sphere on the plane. In the discussion, Guidubaldo also describes some drawing tools for tracking some curves: circumference of large radius, hyperbole, conchoidal and ellipse. For this last curve, the author refers to the most common construction, called "Gardener's Ellipse". It is possible with a fixed-length rope, bound in its extremes. The construction takes place through the use of a tool known as 'elliptical compass'.
The instrument consists of a set square, and a grooved slide ruler, in which two sliders slide bound to the same square. At the end of the ruler, it is placed a tracer stylus. Aligning squaring the ellipse's axes, it is possible to trace the conical for separate quarters, and in the same way is possible to trace concentric and equidistant ellipses.
The movement made by the ruler tracing refers as said to the graphic construction of the cycloid. This curve was studied by Pascal, who in his essay "*Histoire de la Roulette*" (1658) describes the cycloid's geometrical properties, that for the simplicity of its construction could also be applied in large construction sites. Recalling the title of the book, it is necessary to point out that "*roulette*" is the French term which indicates the cycloid, already known to the Romans who called it "*cycloidis*" or "*trochoidis*". The properties of the axes of the above geometric construction allows to orient the position of the vertical section planes, according to these directions. This in order to evaluate and calculate the slope of the three *cavee*, starting from the inclination of the flights that cross them. Against those plans, it was sectioned the mesh model, and it was verified the gradient of ramps crossing the *ima-cavea*, the *media-cavea* and *summa-cavea*. The plans - not through the vertical axis joining the centres of the ellipses - determine not symmetrical sections. Therefore, the analysis of the sections was performed only on the portion that affects the northern quadrant of the amphitheater. The slope is calculated between the horizontal direction and the direction determined by the intersection between the section of the rises and treads. It verified the constant inclinations for *ima, media* and *summa cavea*.
Further geometric analysis involved the determination of planimetric profile of the ambulatory that runs below *media cavea*. This masonry archway communicates with the outside world through the two main entrances of the arena, and through two corridors that pierced the western side. The construction of barrel vaulted ambulatory as well as the masonry stairways, superimposed planes and above the buttresses with blind arches is made possible thanks to the use of the *opus caementicium* and, in particular, for structures with vaulted roof, also by the use of the structure arched.
The realization of large public buildings, such as amphitheaters, was possible through improvement in technology of the vaults during the second century b.C. by the Romans, as well as the replacement of the unstable wooden structure with stable masonry construction. It should be noted that, since the third century b.C., the use of the masonry arch was one of the most important flywheels of Roman architecture development, which reaches its peak with the construction of one of the largest amphitheaters of the imperial age, the *Colosseum*, where you can still see three tiers of arches supporting the auditorium.

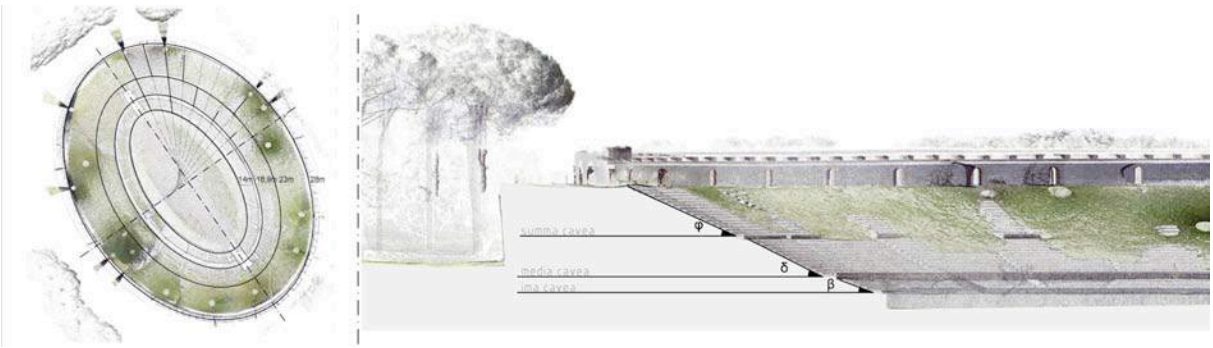

Fig. 6: Geometrical test and evaluation about slope on plan and vertical sections.

This is because the arch, present in many ancient cultures (Egyptian, Babylonian, the Mycenaean, Etruscan) was used by the Romans with consistency and awareness, compared to the ancient contexts in which he was born; roofing systems that utilized both the Egyptians that the Babylonians, are not based on the authentic arches, but on the "pseudo-arches" or "pseudo-vaults" for the nature of the thrusts which arise through the type of roof used. Only the Roman engineers were able to solve the problem of "pushing system", making it a recognizable and essential part of their architecture so much to get to build large vaulted structures like the vault of the Pantheon and the Temple of Diana in Baia.
If we consider that the use of brick arch joins the use of *opus caementicium*, that is a mortar made up of sand and lime, known since the time of the Egyptians and Babylonians but introduced to Europe by the Romans, it is possible understand how the Roman architects were able to take advantage of all types of stone and materials, experiencing endless ways to cut the ashlars and build them, in order to perfect the construction of the vaults. These vaults were built, especially in monumental buildings such as the amphitheater, through the centering, that is a sturdy wooden support tracing exactly the profile of the vertical section of the vaulted roof of the ambulatory.
The ambulatory curve that runs below the *media cavea* has been verified by a similar process described in the previous paragraph. In fact, the application of Pascal's theorem to six randomly chosen points on the profile of the outer wall of the underground corridor, have drawn a hexagon with the opposite sides which intersect with respect to three points aligned in a straight line.

4. The amphitheater shape from the iconography to identification of its geometric surface (Pasquale Argenziano)

The "*Spectacula"* is the largest public building in this city, and for its majesty exceeded in importance compared to the role of an average provincial town, which was Pompeii. The building, in fact, was designed for about 20,000 spectators, of course, considering the inflow of visitors from neighbouring towns, for example among these were the veterans of Sulla, residents in the surrounding area (Avellino or Nola). The current appearance of the amphitheater is the result of various natural and man-made actions, that occurred in the course of approximately 2000 years, from 70 BC (construction date) until the 70s of the Twentieth century, during which the most significant restoration works were completed. During this large period, the amphitheater was severely damaged in the earthquake of 62 A.D. - that strongly hit Pompeii and the Vesuvian region - and the eruption of AD 79.
Some of the damage caused by the earthquake had not yet been repaired at the eruption time. Among the three buildings used for public performances, only the amphitheater was usable again few years from the earthquake, thanks to a consolidation program. The majority of seismic damages of public and private buildings in Pompeii, were repaired using the clay brick, widely available for cheapness and speed of production, besides for the combining flexibility with the blocks of tuff or limestone. In the amphitheater, the clay brick was used mainly for the buttresses and the reinforcing arches of the entrance ambulatory to the arena. The size of the bricks in fact very well matched to model the reinforcement structures of the bowed architectural elements. The interventions conducted by C. Cuspius Pansa (father and son) - after the earthquake - modified the original appearance of the amphitheater, as it is possible see in comparison with the famous fresco representing the fight between Nocerians and Pompeians. This fresco - dated to 59 A.D. - was found in the garden of the house I.3.23. The obvious differences concern the arena parapet, and the series of arches of the summit the *cavea* sector, besides the brick buttresses already mentioned. In the fresco, the arena parapet, appears decorated with faux marble painting, while at the time of the excavation were found some paintings with gladiatorial scenes, which are now kept in copy at the Archaeological Museum of Naples. In the fresco of the "fight", again, the wall summit amphitheater is intended to support the "*maenianum summum in ligneis*". The reconstruction of this wooden elements is visible in the iconographic sequence of the most important buildings of Pompeii, arranged as a frame of the cartography, edited on 1832. This drawing - also enhanced by a panoramic view of the city - was draft by W.B. Clarke, and engraved by T.E. Nicholson for the "Society for the Diffusion of Useful Knowledge Atlas" (1844). This mapping is particularly interesting because it documents the planimetric shape Amphitheatre in detail, with the layout of the four vehicular and pedestrian entrances to the *cavea*, and to the Western underground areas. Following the consolidation work, the Amphitheatre reached its maximum capacity. After the eruption, the building remained visible in its upper part, which was the object of continuous looting, especially as regards the decorative elements and most of the stands.
The damaged architectural elements and their restoration work are documented in most of the iconographic archival repertoire, selected for this research. As already mentioned, the damaged architectural areas are documented in the gouaches, prints and photographs in the nineteenth century. The restoration of the perimeter arches, and the summit walls of the *cavea* are documented in a photographs' collection, dating back to the 60s of last century, preserved in private collections or in the archive of the Superintendent. These images are particularly interesting because they show the

various stages of restoration, from the ruined structures, to the building site (scaffolds were made of wood and non-metallic), till to the building as well as the visitors can still see today.
Considering the seismic and eruptive damages and the destructive action of man in the period after the eruption, the best preserved amphitheater area is the north-eastern quarter, as well as documented through various iconographic documents. The first considerations about the spatial geometry of the stands have been focused precisely in this area of the *summa-cavea.* It was verified that the ramps' axes lean on the elliptical sections at the top and at the base of each cavea, maintaining constant their inclination, respect to the horizontal plane. These arrays of straight lines do not converge (in this case it would be of a conical surface) but they lean on a straight line parallel, belonging to the plane containing the major axes, and parallel to them. The surface generated can be ascribed to the "ruled surfaces" family, being defined by the movement of a generating line leaning on three directrixes: two ellipses and the line, just described. In particular, the surface thus defined is classifiable between the ruled surfaces "non-developable", because two directrixes (in a close neighbourhood) are skewed, and therefore the surface can not be lying on a plane.
The ruled surfaces (ie the surfaces generated by the motion of a straight line) were so named by Jean Nicolas Pierre Hachette, who attributed to them the nickname of "*surfaces réglées*", replacing "gauche" (or "deformed") already used with a derogatory meaning. The variation of the title was used to emphasise the aesthetic value of these particular surfaces. Ruled surfaces were already known to Aristotle that in "De Anima" (I, 4) states, "the motion of a line produces a surface". But we have to wait more than twenty centuries to the publication of a scientific study of these surfaces. In the 60s of the eighteenth century, Gaspard Monge began to study the properties of ruled surface. The study of Monge was published in "Memoire sur les homesteads de plusieurs genres de surfaces courbes, particulièrement sur des cell surfaces développables avec une application a la theorie des ombres et des pénombres". In the text he distinguishes between "developable surface" and "hump ribbed surface", defines their apparent contour in perspective representation, and describes the genesis of the ruled surfaces as a "set of straight lines", that lean three fixed lines. The realisation of the Amphitheatre's 3D model, as a rational geometric extraction from point cloud, will allow the development of digital comparison among the scanned shape, the modelled shape, and the theoretical reconstruction according the classic treatises.

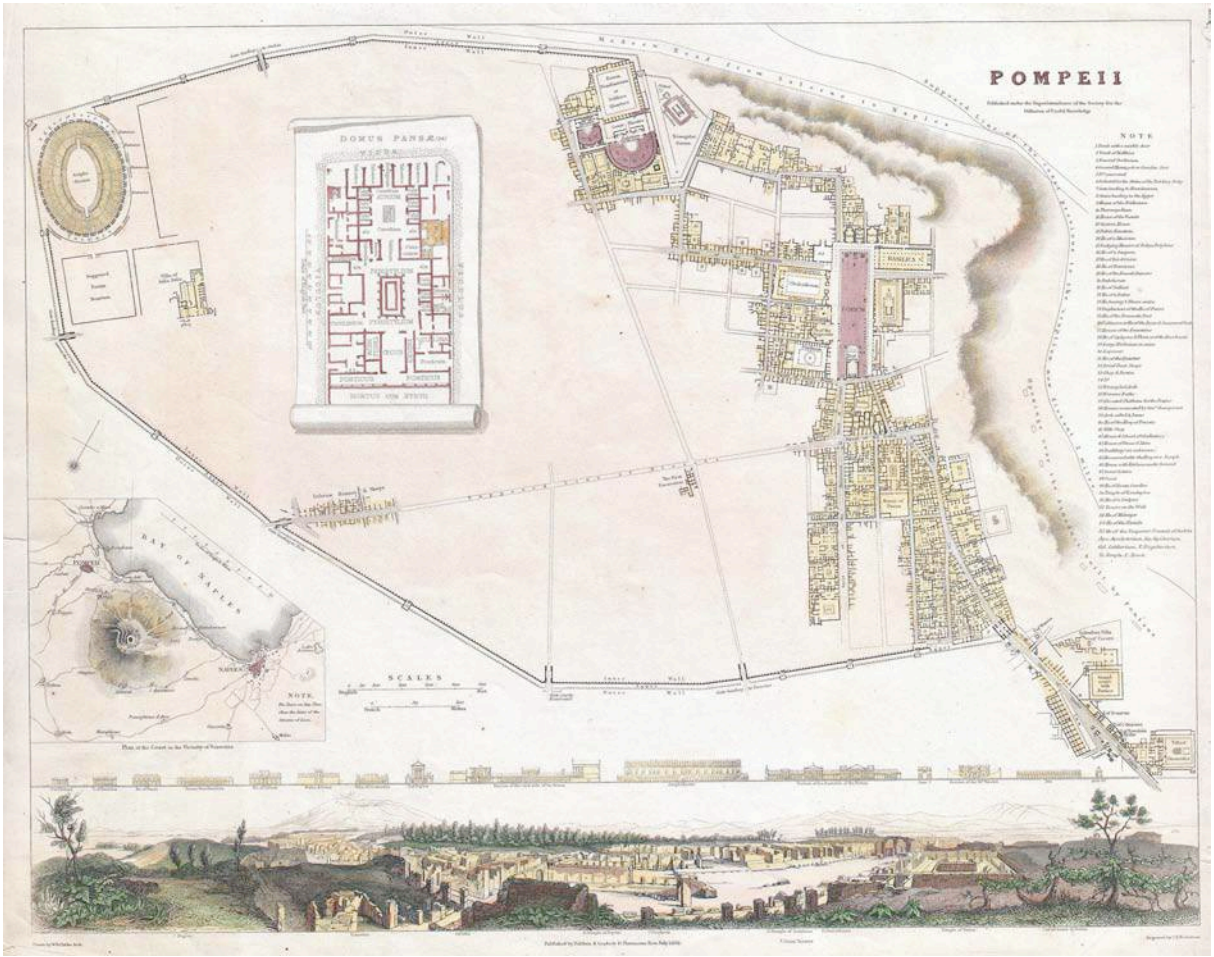

Fig. 7: Pompeii, map and panoramic view of the city - drawing by W.B. Clarke (1832); engraving by T.E. Nicholson for the "Society for the Diffusion of Useful Knowledge Atlas" (1844).

5. Conclusions (Carmine Gambardella, Nicola Pisacane, Alessandra Avella, Pasquale Argenziano)

The geometric investigations described in this paper allowed to reach the determination of the form, which best approximates the amphitheater's *cavea*. The assumptions of projective geometry and its application's outcomes in descriptive geometry, initially allowed to investigate significant plane sections (horizontal and vertical ones according directions suitably chosen) in order to verify: the planimetric shape of the stands and the Hypogea; the geometric rule that guides the direction of the flights towards the different cavea's areas; the calculation of the slope of *ima-cavea, media-cavea* and *summa-cavea* From analysis of the plane sections of the mesh model, it was possible to build 3D models, and define the rules that generate the ruled surfaces enveloping the three areas of the *cavea*.

The so performed geometrical analysis, through selective interrogations of point cloud model, are based on the methodological assumption, that the discrete model is the crystallization of the building to the instrumental capturing date. Given the particularities of the building, and its articulated the architectural and restorative events before and after the eruption of Vesuvius, the research group considered it important to support the geometric analysis with a large iconographic range (gouaches, prints, archival photographs) from whom deducing, in superposition on the reference bibliography, the authenticity or not of the architectural sectors, and therefore the reliability of the position and of the morphology of the various elements. If on one hand this methodological path was valid decision support to the 3D modeling, on the other one it opened up a new investigation item - to develop in future - inherent in the analysis of the various projective iconography in connection to the building.

For example, the pictures - dating from the mid-nineteenth century - will be re-screened in 3D digital scene of the Amphitheatre, in order to calculate the position of the trigger point, and the focal camera used at the time, and therefore it will be possible texturize the point cloud model (and the solid model derived) with the historical images.

This process will allow to display the scene observed by the photographer, and then to implement an accurate dating of the architectural parts, compared to the images' timeline and archaeological events, which affected the building in the last century.

Fig. 8: Integrated data from LIDAR sensors and terrestrial technology.

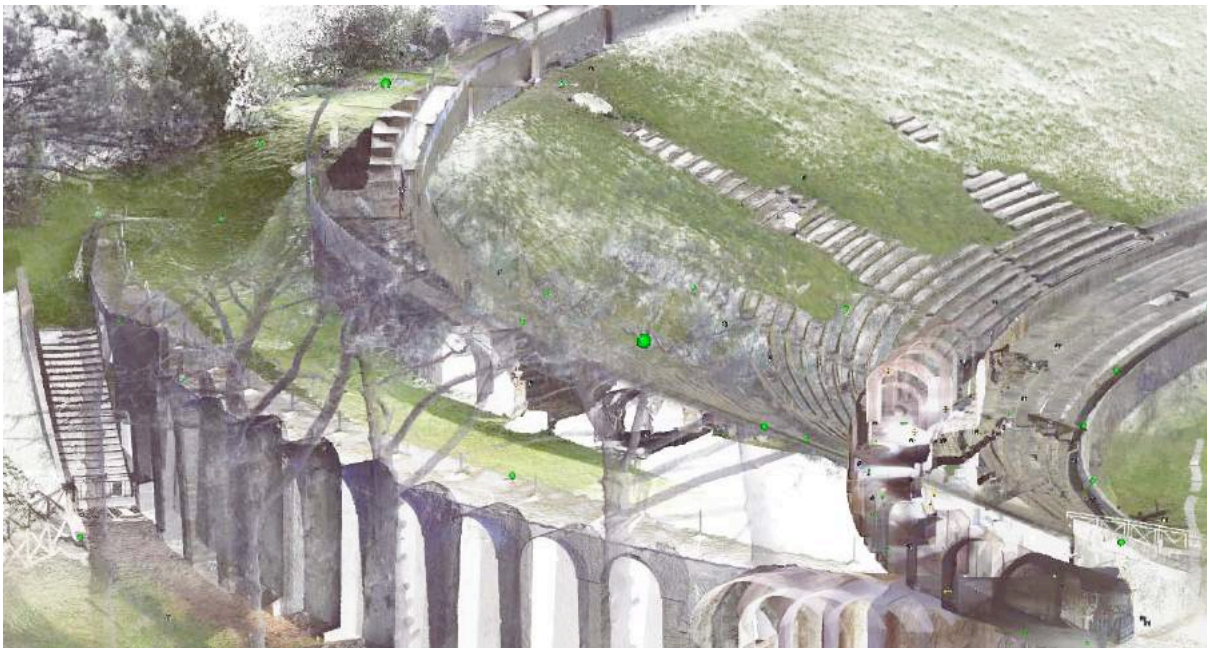

Fig. 9: Pompeii amphitheatre: external, internal and underground simultaneous view from 3D points cloud model.

Bibliography

[1] J.P. Adam, L'arte di costruire presso i Romani, Longanesi, Milano 2008.

[2] G. Alisio, P.A. De Rosa, P.E. Trastulli. Napoli com'era nelle gouaches del Sette e Ottocento. Newton & Compton Editori, Roma, 1990.

[3] L. Cresci. Le curve celebri. Invito alla storia della matematica attraverso le curve piane più affascinanti. Mondolibri, Milano, 1998.

[4] M. Dell'Aquila. Il luogo della geometria. Arte Tipografica, Napoli, 1999.

[5] R. de Rubertis. Un enigma avvincente. Il tracciato planimetrico ellittico del Colosseo. *Disegnare. Idee Immagini*, 1999.18/19:99-105.

[6] S. Duvernoy and P. L. Rosin. The Compass, the Ruler and the Computer: An Analysis of the Design of the Amphitheatre of Pompeii. In K. Williams and M. J. Ostwald (ed.) *Architecture and Mathematics from Antiquity to the Future. Volume II: The 1500s to the Future*, pages 525-540. Birkhäuser-Springer, Heidelberg, 2015.

[7] C. Gambardella, N. Pisacane, A. Avella, P. Argenziano. Multisensor and multiscale surveying into Pompeii's archeological site. Three case studies. In C. Gambardella *Le Vie dei Mercanti. Heritage anf Technology. Mind Knowledge Experience,* pp. 2429-2468. La Scuola di Pitagora, Napoli, 2015.

[8] C. Gambardella, N. Pisacane, P. Argenziano, A. Avella. Rilievo digitale integrato dell'anfiteatro nel sito archeologico di Pompei. Integrated digital survey of the amphitheatre in the archaeological site of Pompeii. In *Abitare la Terra. Dwelling on Earth*, 2015.38:32-37.

[9] A. Giordano. Cupole volte e alter superfici. La genesi e la forma. UTET, Torino, 1999.

[10] P. Gros, L'architettura romana dagli inizi del III secolo a.C. alla fine dell'alto impero: i monumenti pubblici, Longanesi, Milano 1996.

[11] D. Jacazzi, 'Pompei e la "valle diruta". Tracce e memorie della città antica nelle fonti medioevali e moderne'. In C. Gambardella (ed.) *Atlante di Pompei*, pp. 65–80. La Scuola di Pitagora, Napoli, 2012.

[12] R. Migliari. Ellissi e ovali. Epilogo di un conflitto. *Palladio. Rivista di Storia dell'Architettura e Restauro*, 1995.16:93-102.

[13] F. Pesando, M.P. Guidobaldi, Pompei, Oplontis, Ercolano; Stabiae. Guide Archeologiche Laterza, Roma 2006.

[14] P. Rosin and E. Trucco. The Amphitheatre Construction Problem. In *Incontro Internazionale di Studi Rileggere l'Antico*, pages 1-10, Roma 2005.

[15] R. Salati. La geometria degli anfiteatri romani. Verifica delle ipotesi di tracciamento applicate ai più antichi esempi presenti in area campana. Verlag Editore, Saarbrücken, 2014.

[16] G. Tosi, Gli edifici per spettacoli nell'Italia romana, I, Quasar, Roma 2003.

[17] V. Valerio. Sul disegno e sulla forma degli anfiteatri. *Disegnare. Idee Immagini*, 1993.6:25-34.

[18] P. Zanker, *POMPEI*, Einaudi, Torino 1993.

[19] O. Zerlenga. La <<forma ovata>> in architettura. Rappresentazione geometrica. CUEN, Napoli, 1997.

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